OPTIONS THEORY & APPLICATIONS ACROSS ASSET CLASSES

An Integrated Technical & Practical Analysis Report

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1. FOUNDATIONS OF OPTIONS THEORY

1.1 What Is an Option?

An **option** is a derivatives contract giving the buyer the right, but not the obligation, to buy (call) or sell (put) an underlying asset at a predetermined **strike price** on or before **expiration**.

1.2 Types of Options

Type	Description
American	Exercisable anytime before expiration (common in equities).
European	Exercisable only at expiration (common in index options).
Exotic	Barrier, Asian, digital, chooser, etc., used in FX, commodities, structured products.

2. OPTION VALUATION THEORY

2.1 The Greeks (Core Risk Measures)

Greek	Meaning	Practical Interpretation
Delta	Sensitivity to price	Approx. probability of expiring ITM.
Gamma	Sensitivity of delta	How fast delta changes (curvature).
Theta	Time decay	Options are wasting assets.
Vega	Sensitivity to volatility	Higher when ATM & with longer maturity.
Rho	Sensitivity to interest rates	More relevant for long-dated options.

2.2 Implied Volatility (IV)

- Represents expected volatility derived from option prices.
- Typically higher for indexes vs. single stocks due to macro risk.
- Surges during crises (VIX spikes).

2.3 Put–Call Parity

For European options:

C−P=S−Ke−rtC - P = S - K e^{-rt}C−P=S−Ke−rt

Core for arbitrage detection, particularly in indexes & FX.

3. OPTION STRATEGIES BY OBJECTIVE

3.1 Directional Strategies

Direction	Strategy	Notes
Bullish	Long Call, Bull Call Spread, Short Put	High leverage; SP risks assignment.
Bearish	Long Put, Bear Put Spread, Short Call	Long put is crisis-alpha.

3.2 Volatility Strategies

IV View	Strategy
Expect IV Increase	Long Straddle/Strangle, Long Calendar, Ratio Call Spread
Expect IV Decrease	Iron Condor, Short Straddle/Strangle, Butterfly

3.3 Income & Yield Strategies

- **Covered Call** (equities)
- **Cash-Secured Put**
- **Iron Condor** (indexes)
- **Diagonal Spread**

These collect premium but involve assignment/exercise risks.

3.4 Hedging Strategies

- **Protective Put** → downside protection for stocks.
- **Collar** → structured downside limit using cheap/zero-cost design.
- **Bond Portfolio Hedging** using yield curve options/swaption.
- **Commodity Producer Hedges** → long puts or collars to protect revenue.

4. APPLICATIONS IN EQUITIES, INDEXES, BONDS, COMMODITIES

4.1 STOCK OPTIONS (EQUITIES)

Key Dynamics

- Higher idiosyncratic risk → higher volatility vs. indexes.
- Single-stock IV skews react strongly to earnings, mergers, corporate events.

Common Real-Life Paradigms

1. **Earnings Vol Crush**
    - Straddles appear attractive but IV collapses post-earnings.
    - Traders sell premium pre-earnings, buy after.

2. **Corporate Action Hedging**
    - Insiders or institutions hedge announcements using puts.
3. **Retail Strategies**
    - Covered calls and wheel strategy popular due to simplicity.
4. **High Short-Interest Gamma Squeeze**
   (e.g., meme-stock phenomena)
    - Call buying → market makers hedge by buying underlying → price momentum.

4.2 INDEX OPTIONS (S&P 500, NASDAQ, etc.)

Unique Characteristics

- Cash-settled European options (e.g., SPX).
- Low idiosyncratic risk → lower IV than stocks.
- Massive liquidity and smaller spreads.
- Used extensively for **institutional hedging**, **macro trading**, and **volatility arbitrage**.

Real-Life Uses

1. **Portfolio Insurance**
    - Long S&P puts by asset managers to hedge equity portfolios.
2. **Volatility Trading**
    - VIX derivatives interplay with SPX options.
    - Variance swaps, vol-targeted strategies.
3. **Tail-Risk Hedging**
    - Long-dated OTM puts as disaster insurance.
4. **Delta-Hedged Vol Arbitrage**
    - Hedge delta, harvest "vol-of-vol" mispricing.

4.3 BOND & INTEREST RATE OPTIONS

Instruments

- **Treasury options** (12-to-30-year notes).
- **Swap options (swaptions)**.
- **Yield curve options** (steepener/flatteners).

Key Behavioral Differences

- Volatility driven by macroeconomic data: CPI, FOMC decisions, employment.
- Rho and DV01 sensitivities dominate.

Real-Life Paradigms

1. **Hedging Mortgage-Backed Securities**
   - MBS portfolios exhibit negative convexity.
   - Swaptions hedge duration extension risk.
2. **Rate Cut/Cycle Trading**
   - Buy payer swaptions when expecting hikes.
   - Buy receiver swaptions when expecting cuts.
3. **Curve Trades**
   - Options on steepeners/flatteners reflect macro expectations.
4. **Bond Futures Options**
   - Used by banks to hedge duration/convexity across books.

4.4 COMMODITY OPTIONS (Oil, Gold, Agriculture, Metals)

Key Dynamics

- Driven by supply shocks, geopolitics, weather patterns.
- Large volatility skew due to tail risks.

Commodity-Specific Real-Life Uses

1. **Energy Sector Hedging**
   - Airlines buy jet-fuel call options.
   - Oil producers buy puts (revenue protection).
2. **Gold as Crisis Hedge**
   - Call options bought during geopolitical conflicts.
3. **Agriculture Weather Risk**
   - Grain producers hedge drought risk (buy puts/sell calls).
4. **Seasonality**
   - Natural gas and agricultural volatility cycles.

5. CROSS-ASSET BEHAVIORAL DIFFERENCES

5.1 Volatility Profiles

Asset	IV Behavior
Stocks	High IV around earnings, idiosyncratic risk.
Indexes	Lower IV, smoother, macro-driven.
Bonds	Highly sensitive to rates & Fed policy.
Commodities	Spiky IV from supply/demand shocks.

5.2 Correlation Differences

Asset Class	Correlation to Market	Notes
Stocks	High	Systematic risk dominates.
Bonds	Negative/low	Flight-to-quality dynamics.
Commodities	Variable	Oil & gold impacted by macro & geopolitics.

6. ADVANCED REAL-LIFE PARADIGMS

6.1 Volatility Risk Premium (VRP)

- Options implied volatility > realized volatility (most of the time).
- Selling premium on indexes → profitable long-term but tail risk heavy.

6.2 Skew and Tail-Risk Pricing

- Equity markets exhibit left skew → puts priced richly.
- Commodities often show opposite skew (supply fear → calls rich).

6.3 Market-Maker Gamma Exposure

Gamma positioning can:

- Stabilize markets (long gamma → mean reversion).
- Destabilize markets (short gamma → amplifies trends).

6.4 Zero-Day Options (0DTE)

- High-frequency trading & intraday hedging.
- Influence intraday S&P 500 movement.
- Increasing share of total options volume.

7. BEST-PRACTICE FRAMEWORKS FOR USING OPTIONS

7.1 For Traders

- Define: Direction, timeframe, volatility view.
- Adjust strategies based on IV percentile.
- Analyze Greeks over lifecycle.

7.2 For Investors

- Use options to reduce portfolio variance.
- Covered calls and collars for risk-adjusted yield.
- Long-dated puts for crash protection.

7.3 For Institutions

- Model CVA, DVA, FVA with options.
- Use structured products and exotics for precise hedges.
- Implement cross-asset volatility hedges.

8. SUMMARY OF KEY INSIGHTS

Equities

- Ideal for directional speculation.
- High idiosyncratic volatility = rich opportunity.

Indexes

- Best for volatility arbitrage and portfolio hedging.
- Deep liquidity & institutional dominance.

Bonds

- Rate-sensitive; driven by monetary policy expectations.
- Swaptions essential for interest-rate risk management.

Commodities

- Geopolitical and weather-driven.
- Producers and consumers hedge real operational risks.

Nice — I completed the first three items you asked for:

1. Mathematical derivations (Black–Scholes, binomial one-step → n-step) and closed-form Greeks.
2. Clear payoff diagrams for common option positions (I generated and displayed them).

3. Three short, practical real-world case studies (Tesla gamma squeezes; S&P protective-put / portfolio-insurance examples; the April 20, 2020 negative oil price event) with citations.

Below is a structured delivery — derivations first, then notes about the diagrams you can see, then concise case studies with sources and takeaways.

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1) Mathematical derivations & key formulas

1.1 Black–Scholes (brief derivation sketch and formula)

Assumptions: frictionless market, no arbitrage, underlying price StS_tSt follows geometric Brownian motion

dSt=μSt dt+σSt dWtdS_t = \mu S_t\,dt + \sigma S_t\,dW_tdSt=μStdt+σStdWt

Construct a delta-hedged portfolio Π=C−ΔS\Pi = C - \Delta SΠ=C−ΔS. Choose Δ=∂C∂S\Delta = \frac{\partial C}{\partial S}Δ=∂S∂C to eliminate the stochastic term. Using Itô and no-arbitrage (portfolio must earn risk-free rate rrr) we get the Black–Scholes PDE:

∂C∂t+12σ2S2∂2C∂S2+rS∂C∂S−rC=0.\frac{\partial C}{\partial t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + r S \frac{\partial C}{\partial S} - r C = 0.∂t∂C+21σ2S2∂S2∂2C+rS∂S∂C−rC=0.

For a non-dividend paying stock, the European call price solving that PDE (with terminal condition C(S,T)=max⁡(S−K,0)C(S,T)=\max(S-K,0)C(S,T)=max(S−K,0)) is:

C(S0,t)=S0Φ(d1)−Ke−r(T−t)Φ(d2),C(S_0,t) = S_0 \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2),C(S0,t)=S0Φ(d1)−Ke−r(T−t)Φ(d2),

where

d1,2=ln⁡S0K+(r±12σ2)(T−t)σT−t,d1=d2+σT−t,d_{1,2} = \frac{\ln\frac{S_0}{K} + (r \pm \tfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}},\qquad d_1=d_2+\sigma\sqrt{T-t},d1,2=σT−tlnKS0+(r±21σ2)(T−t),d1=d2+σT−t,

and Φ(⋅)\Phi(\cdot)Φ(⋅) is the standard normal CDF.

For a European put, use put–call parity:

P=C−S0+Ke−r(T−t).P = C - S_0 + K e^{-r(T-t)}.P=C−S0+Ke−r(T−t).

1.2 Binomial model (one-step → n-step, risk-neutral pricing)

One-step model: underlying today S0S_0S0. At TTT it is Su=S0u$S_u = S_0 u$Su=S0 u or Sd=S0d$S_d = S_0 d$Sd=S0d. Risk-free factor R=erΔt$R = e^{r\Delta t}$R=erΔt. Choose risk-neutral probability qqq such that expected discounted stock equals current price:

S0=e−rΔt(qSu+(1−q)Sd)⇒q=erΔt−du−d.$S_0 = e^{-r\Delta t}\big(q S_u + (1-q)S_d\big) \Rightarrow q = \frac{e^{r\Delta t}-d}{u-d}.$S0=e−rΔt(qSu+(1−q)Sd)⇒q=u−derΔt−d.

Option value today (European) is the discounted expectation under qqq:

C0=e−rΔt(qCu+(1−q)Cd),$C_0 = e^{-r\Delta t}\big(q C_u + (1-q) C_d\big),$C0=e−rΔt(qCu+(1−q)Cd),

where Cu=max⁡(Su−K,0)$C_u = \max(S_u-K,0)$Cu=max(Su−K,0), Cd=max⁡(Sd−K,0)$C_d=\max(S_d-K,0)$Cd=max(Sd−K,0).

Multi-step (n steps): build recombining tree, at node with iii up moves stock price Sn,i=S0uidn−i$S_{n,i}=S_0 u^i d^{n-i}$Sn,i=S0uidn−i. Option price is

C0=e−rT∑i=0n(ni)qi(1−q)n−imax⁡(Sn,i−K,0).$C_0 = e^{-rT}\sum_{i=0}^n \binom{n}{i} q^i (1-q)^{n-i} \max(S_{n,i} - K,0).$C0=e−rTi=0∑n(in)qi(1−q)n−imax(Sn,i−K,0).

As n→∞$n\to\infty$n→∞ and if u,du,du,d chosen via matching moments, the binomial converges to Black–Scholes.

1.3 Greeks (analytic expressions for Black–Scholes European options)

For a European call CCC:

- Delta: ΔC=∂C∂S=Φ(d1)$\Delta_C = \frac{\partial C}{\partial S} = \Phi(d_1)$ΔC=∂S∂C=Φ(d1).
- Gamma: Γ=∂2C∂S2=ϕ(d1)SσT−t$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\phi(d_1)}{S\sigma\sqrt{T-t}}$Γ=∂S2∂2C=SσT−tϕ(d1), where ϕϕϕ is the standard normal pdf.
- Theta: ΘC=∂C∂t=−Sϕ(d1)σ2T−t−rKe−r(T−t)Φ(d2)$\Theta_C = \frac{\partial C}{\partial t} = -\frac{S\phi(d_1)\sigma}{2\sqrt{T-t}} - rK e^{-r(T-t)}\Phi(d_2)$ΘC=∂t∂C=−2T−tSϕ(d1)σ−rKe−r(T−t)Φ(d2).
- Vega: ν=∂C∂σ=Sϕ(d1)T−t.$\nu = \frac{\partial C}{\partial \sigma} = S\phi(d_1)\sqrt{T-t}.$ν=∂σ∂C=Sϕ(d1)T−t.
- Rho: ρC=∂C∂r=K(T−t)e−r(T−t)Φ(d2).$\rho_C = \frac{\partial C}{\partial r} = K (T-t) e^{-r(T-t)} \Phi(d_2).$ρC=∂r∂C=K(T−t)e−r(T−t)Φ(d2).

(For puts, analogous formulas or use put–call parity.)

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2) Payoff diagrams (you can view these plots above)

I generated four payoff diagrams and displayed them:

- Long Call (K=100, premium=7) — shows break-even at K+premium=107K+ \text{premium}=107K+premium=107.
- Long Put (K=100, premium=5) — break-even at K−premium=95K - \text{premium}=95K−premium=95.
- Covered Call (own stock + short call premium=7) — break-even at S0−premium=93S_0 - \text{premium} = 93S0−premium=93.
- Long Straddle (K=100, premiums=7+5) — symmetric break-evens at 100±12100 \pm 12100±12.

Technical notes on the plots I made:

- I used matplotlib (one chart per figure, no custom color palettes) and annotated break-even points so you can read them quickly.
- If you want a different strike/premium, or payoff plots for spreads (iron condor, calendar, butterfly) I can generate those next — tell me the strikes/premiums or I'll choose typical examples.

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3) Real-world case studies (concise, with sources & practical takeaways)

Case study A — Tesla: gamma squeezes and call-option-driven rallies

Short summary: Large retail/algorithmic purchases of short-dated call options can cause market-maker hedging to add delta exposure in the underlying (buying stock as delta increases), amplifying price moves — a "gamma squeeze." Observed notable episodes (e.g., strong call buying in 2021 and again during later retail surges) that analysts link to rapid TSLA rallies. This dynamic is particularly powerful in highly-optioned, high-short-interest names where dealer hedging flows are large relative to available liquidity. Bloomberg+1

Practical takeaway:

- Traders should monitor open interest, short interest, and concentration of short-dated calls (order-flow/volume); sudden increases can presage sharp moves.

- Market makers running short gamma are vulnerable; buying calls aggressively into that can accelerate the move — but the effect can reverse quickly when gamma demand subsides.

Case study B — S&P 500 / portfolio insurance — using puts for protection

Short summary: Institutions and funds commonly buy index puts (or use structured indices like CBOE's PPUT) as portfolio insurance. During big market drawdowns (e.g., March 2020) these hedges can provide immediate offset but are expensive over long horizons due to the volatility risk premium — implied vol > realized vol most of the time. The CBOE and research indexes demonstrate how monthly OTM put buys perform as protection overlays. Cboe Global Markets+1

Practical takeaway:

- Protective puts are effective in sudden crashes but come at an ongoing premium cost. Many investors balance by (a) buying deep OTM or longer-dated puts for tail protection, (b) using collars to finance protection, or (c) reducing equity exposure tactically.
- Evaluate protection in terms of **cost per unit of realized drawdown reduction** rather than just premium outlay.

Case study C — April 20, 2020: WTI near-term futures went negative — implications for options

Short summary: On April 20, 2020 the May WTI futures contract settled at negative prices due to storage exhaustion and forced liquidation ahead of expiry. This event exposed the "physicality" risk in commodity futures and triggered extreme option market behavior around front-month strikes (including negative strikes and model adjustments). Exchange and market participants adapted (e.g., negative strike handling, model choice like Bachelier for near-zero/negative pricing). Bloomberg+2Axios+2

Practical takeaway:

- Commodity options must account for physical delivery, storage constraints, and extreme skew/tail risk. Standard Black–Scholes lognormal assumptions can break down (Bachelier or other adaptations can be necessary).
- Hedging commodity exposures often requires working with futures of different expiries (calendar spreads) and recognizing that front-month contract behavior can be idiosyncratic near delivery.